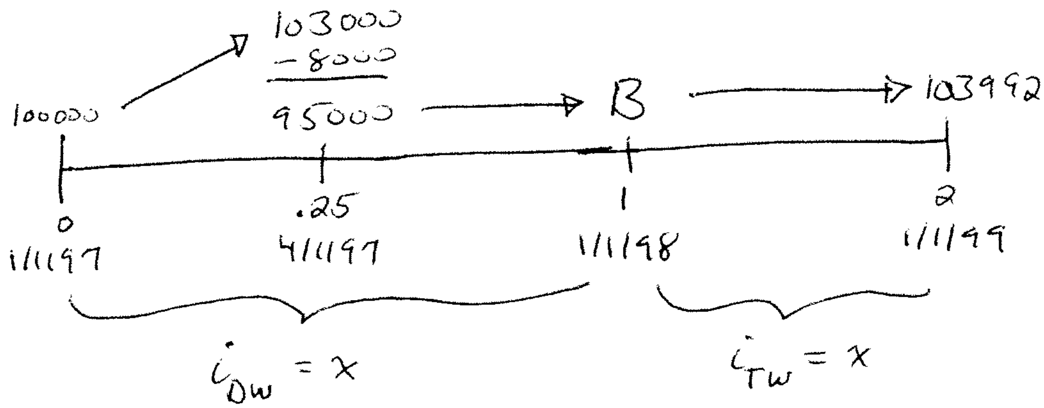
16 / May 2000 Course 2 Exam



For 1997: $100000(1+x) - 8000(1+.75x) = B$ (1)

For 1998: $1+x = \dfrac{103992}{B}$ (2)

$$(1) \Rightarrow 92000 + 94000x = B$$

$$(2) \Rightarrow B = \frac{103992}{1+x}$$

$$\therefore 92000 + 94000x = \frac{103992}{1+x}$$

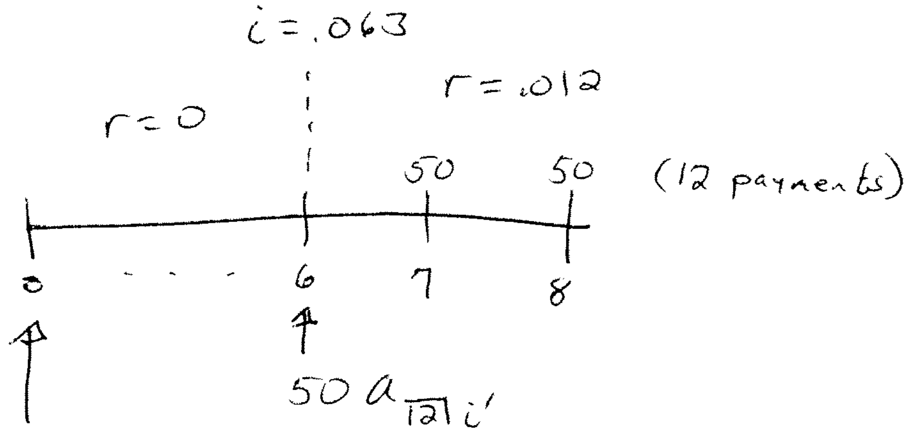
$$\Rightarrow (1+x)(92000 + 94000x) = 103992$$

$$\therefore 94000x^2 + 186000x - 11992 = 0$$

$$\Rightarrow x = \frac{-186000 + \sqrt{(186000)^2 + 4(94000)(11992)}}{2(94000)}$$

$$\doteq .0625$$

51 / May 2000 Course 2 Exam



PV = X

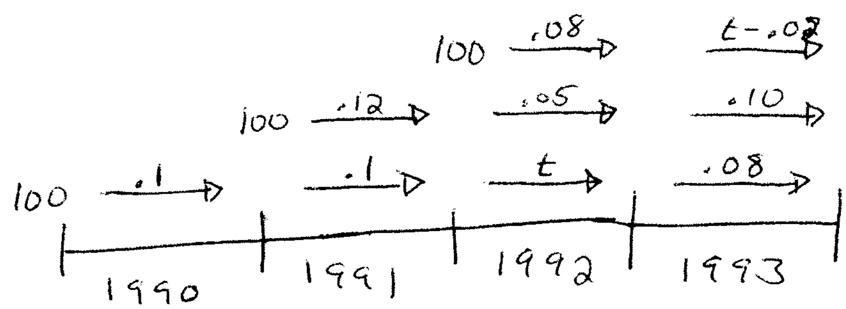
$$i' = \text{real rate of return} = \begin{cases} .063 & 0 \le t \le 6 \\ \frac{1.063}{1.012} - 1 & t > 6 \end{cases}$$

$$1 + i' = \frac{1+i}{1+r}$$

$$\therefore X = PV = 50\, a_{\overline{12}|\left(\frac{1.063}{1.012}-1\right)} \cdot v_{.063}^{6}$$

$$\doteq 306.48$$

2/ Nov. 2000 Course 2 Exam) The amount of interest credited during 1993 equals the balance at the end of 1992 times the credited interest rate for 1993.

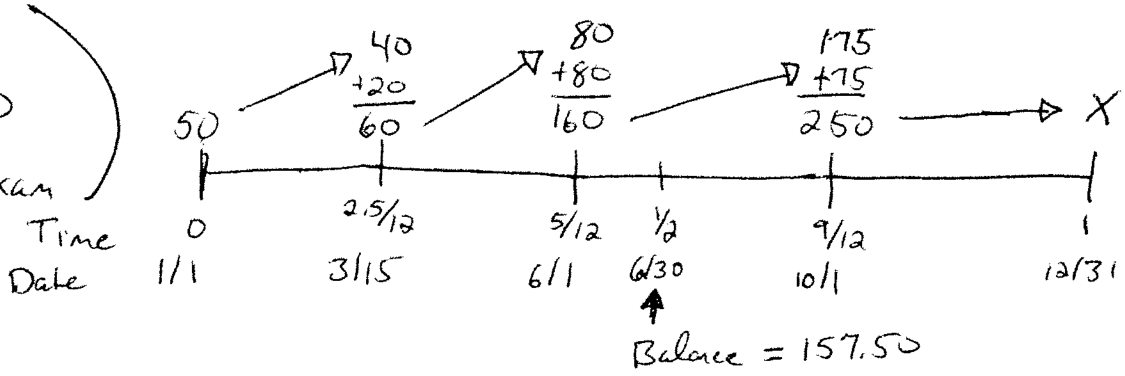


$$\therefore\ 28.40 = 100(1.1)(1.1)(1+t)(.08) + 100(1.12)(1.05)(.1) + 100(1.08)(t-.02)$$

$$\therefore\ 28.40 = 9.68(1+t) + 11.76 + 108(t-.02)$$

$$\Longrightarrow t \doteq .0775$$

27 / Nov. 2000 Course 2 Exam



Using TW, the annual yield for the first 6 months is determined by:

$$(1+i)^{1/2} = \frac{40}{50} \cdot \frac{80}{60} \cdot \frac{157.50}{160} = 1.05$$

$$\Longrightarrow i = 10.25\%$$

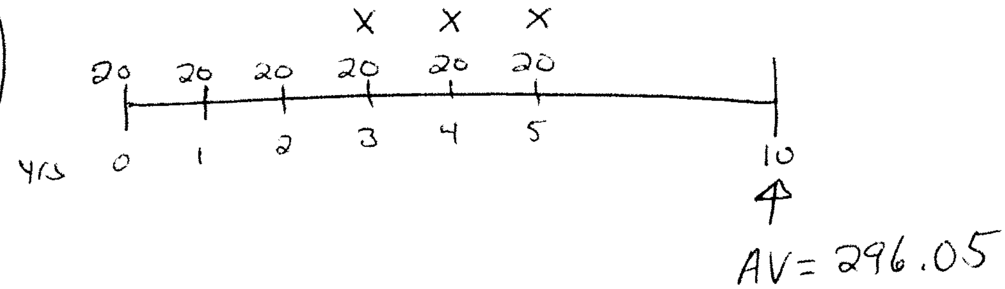
$\therefore$ the TW annual effective yield for the entire year is 10.25%

$$\therefore \quad 1.1025 = \frac{40}{50} \cdot \frac{80}{60} \cdot \frac{175}{160} \cdot \frac{X}{250}$$

$$\Longrightarrow X = 236.25$$

38 / Nov. 2000 Course 2 Exam

Chuck needs $200(1.04)^{10} = 296.65$ in 10 years.



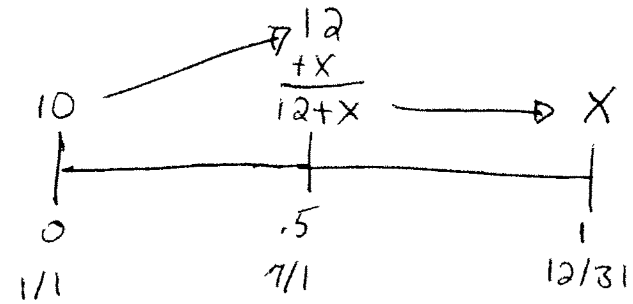
Chuck needs 296.05 at time $t=10$. His investments earn 10% aeir. We don't use the real rate of return when accumulating his deposits, since we've already accounted for inflation when determining the amount Chuck needs at time $t=10$.

The equation we solve is:

$$296.05 = \underbrace{20\, s_{\overline{6}|.1}(1.1)^5 + X\, s_{\overline{3}|.1}(1.1)^5}_{\text{we could have used } \ddot{s}\text{'s if we wanted.}}$$

$$\Longrightarrow X \doteq 8.9$$

31 / May 2001 Course 2 Exam



$$i_{TW} = 0 \Rightarrow 1+0 = \frac{12}{10}\cdot\frac{X}{12+X} \Rightarrow 1 = \frac{1.2X}{12+X}$$

$$\Rightarrow 12+X = 1.2X \Rightarrow X = 60$$

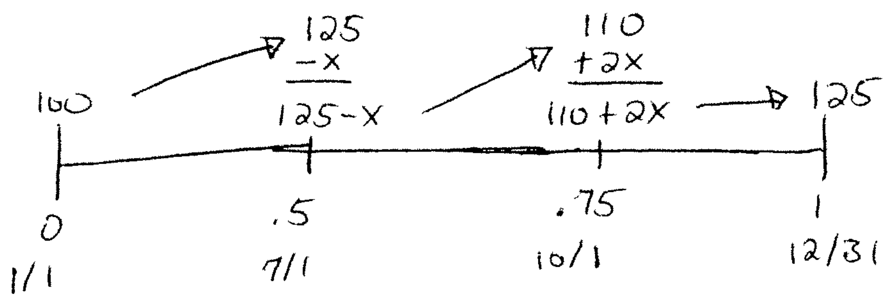
$$i_{DW} = Y \Rightarrow 10(1+Y) + \overset{60}{X}\left(1+\tfrac{1}{2}Y\right) = \overset{60}{X}$$

$$\Rightarrow 70 + 40Y = 60$$

$$\Rightarrow Y = -0.25$$

20 / Nov. 2001 Course 2 Exam

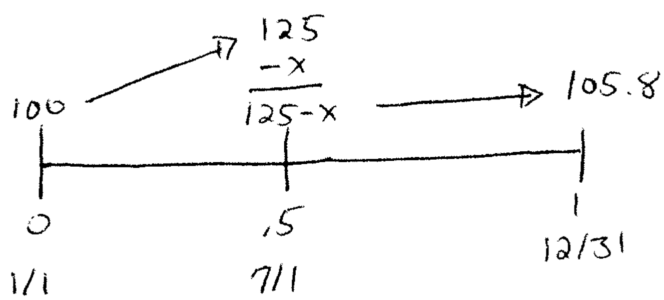
Account K:



(1)

$i_{DW} = i \Rightarrow 100(1+i) - X(1+.5i) + 2X(1+.25i) = 125$

Account L:



(2)

$i_{TW} = i \Rightarrow 1+i = \frac{125}{100} \cdot \frac{105.8}{125-X} \Rightarrow (1+i)(125-X) = 132.25$

Solve (1) and (2) simultaneously. (Many way!)

Let's solve for X in (1) and substitute into (2).

From (1): $100 + 100i - X - .5Xi + 2X + .5Xi = 125$

$\Rightarrow X = 25 - 100i$

$\therefore$ by (2), $(1+i)(125-(25-100i)) = 132.25$

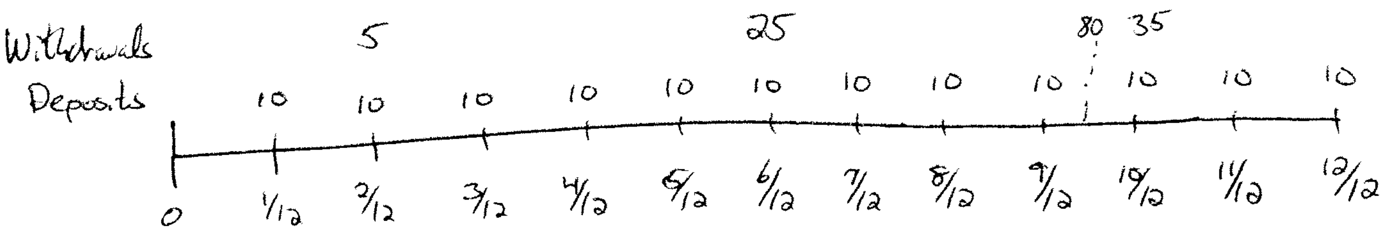
$\Rightarrow (1+i)(100+100i) = 132.25$

$\Rightarrow 100(1+i)^2 = 132.25 \Rightarrow i = 15\%$

17 / (May 2003 Course 2 Exam)

$A = \text{BOY Amount} = 75$

$B = \text{EOY Amount} = 60$



$i_{DW} = i \implies$

$$75(1+i) + 10\left(1+\tfrac{11}{12}i\right) + 10\left(1+\tfrac{10}{12}i\right) + \cdots + 10$$

$$-5\left(1+\tfrac{10}{12}i\right) - 25\left(1+\tfrac{6}{12}i\right) - 80\left(1+\tfrac{2.5}{12}i\right) - 35\left(1+\tfrac{2}{12}i\right) = 60$$

$$\therefore 75 \overset{+75i}{+} 10(12) + \frac{10}{12}i\overbrace{(11+10+9+\cdots+1)}^{= \frac{n(n+1)}{2} = \frac{11(12)}{2} = 66}$$

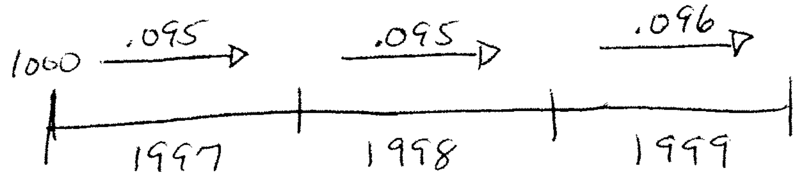
$$-5-25-80-35 - \frac{i}{12}\left(50 + 6(25) + 80(2.5) + 35(2)\right) = 60$$

$$\therefore 75 \overset{+75i}{+} 120 + 55i - 145 - \frac{470}{12}i = 60$$

$$\implies i \doteq 11\%$$

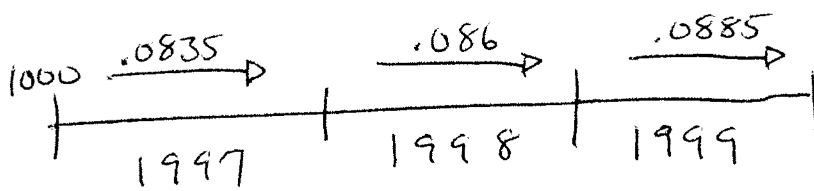
30 / May 2003 Course 2 Exam

P:



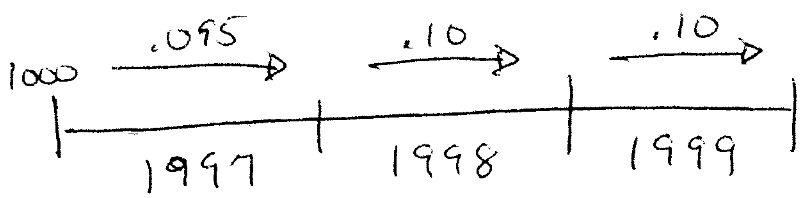
$$P = 1000(1.095)^2(1.096)$$

Q:



$$Q = 1000(1.0835)(1.086)(1.0885)$$

R:

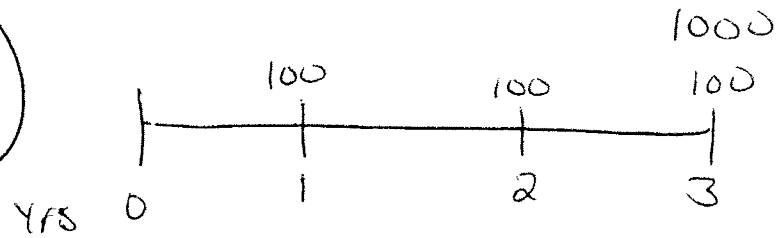


$$R = 1000(1.095)(1.1)^2$$

$$\therefore \quad R > P > Q$$

3/ May 2005 Course FM Exam

$i = .2 = \text{aeir}$



$$X = \text{Mac}\,D = \frac{100v + 100(2)v^2 + 100(3)v^3 + 1000(3)v^3}{100v + 100v^2 + 100v^3 + 1000v^3}$$

$$= \frac{100(Ia)_{\overline{3}|} + 3000v^3}{100a_{\overline{3}|} + 1000v^3} \doteq 2.70$$

6/ (May 2005 Course FM Exam)

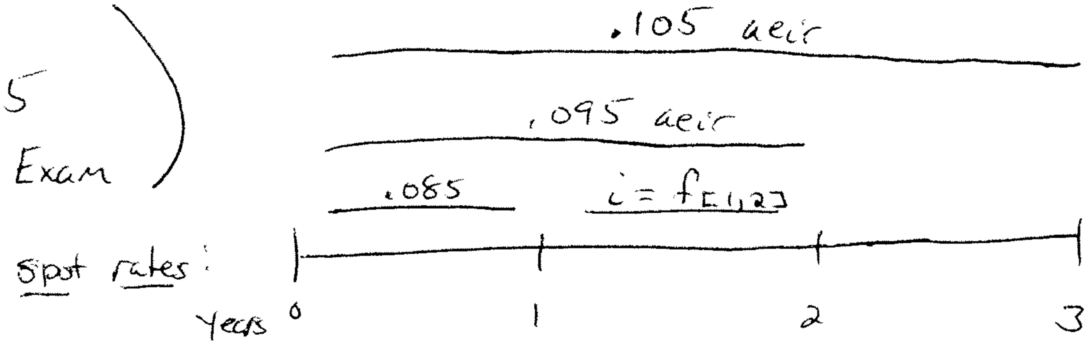
Total Portfolio Price

$= 980 + 1015 + 1000 = 2995$

$$MacD_{Portfolio} = \left(\frac{980}{2995}\right)(21.46) + \left(\frac{1015}{2995}\right)(12.35) + \left(\frac{1000}{2995}\right)(16.67)$$

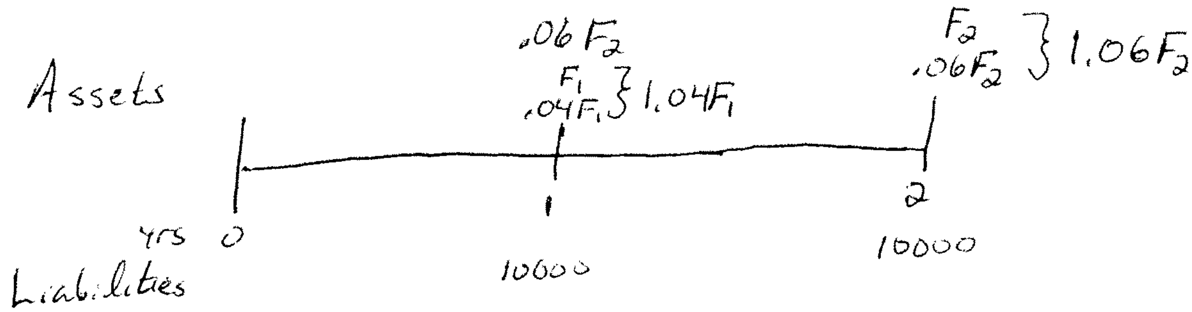
$$\doteq 16.77 \text{ years}$$

10 / May 2005 Course FM Exam



$$(1.095)^2 = (1.085)(1+i) \implies i \doteq 10.5\%$$

15/ May 2005 Course FM Exam) Let F_1 = face amount of the 1-year bond and F_2 = ________________ 2-year bond



Exact Matching $\Longrightarrow \begin{cases} 1.04F_1 + .06F_2 = 10000 \\ 1.06F_2 = 10000 \end{cases} \Longrightarrow \begin{array}{l} F_1 \doteq 9071.1176 \\ F_2 \doteq 9433.9623 \end{array}$

Prices: $P_1 = .04F_1\, a_{\overline{1}|.05} + F_1 v_{.05} = 1.04F_1 \cdot v_{.05} \doteq 8984.73$

and $P_2 = .04F_2\, a_{\overline{2}|.05} + F_2 v^2_{.05} = .06F_2 \cdot v_{.05} + 1.06F_2 v^2_{.05} \doteq 9609.38$

$\therefore$ total cost $= X = P_1 + P_2 \doteq 18594$

Notes: 1) In reality, the 1-year bond has a fixed face value, C_1, and we are purchasing an unknown number, N_1, of 1-year bonds. In the manual we showed this is equivalent to purchasing one 1-year bond with an unknown face value F_1 $(F_1 = C_1 N_1)$. Likewise for the 2-year bond.

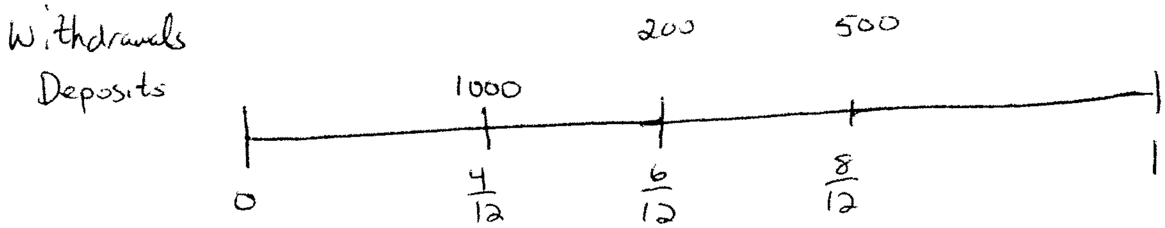
2) $X = P_1 + P_2 = 1.04F_1 v + .06F_2 \cdot v + 1.06F_2 \cdot v^2$.

The v in the first term uses the yield on the 1-year bond whereas the v's in the last two terms use the yield on the 2-year bond. Since the yields are the same in this problem, we get $X = (1.04F_1 + .06F_2) \cdot v + 1.06F_2 \cdot v^2 = 10000v_{.05} + 10000v^2_{.05}$; we didn't need to find F_1 and F_2, since the yields were equal.

16 / May 2005 Course FM Exam

A = BOY Amount = 1000

B = EOY Amount = 1560



$i_{DW} = i$

$$\therefore 1000(1+i) + 1000\left(1+\tfrac{8}{12}i\right) - 200\left(1+\tfrac{6}{12}i\right) - 500\left(1+\tfrac{4}{12}i\right) = 1560$$

$$\Longrightarrow i \doteq 18.57\%$$

Alternative Solution

$A = 1000$

$B = 1560$

C = net total contributions $= 1000 - 200 - 500 = 300$

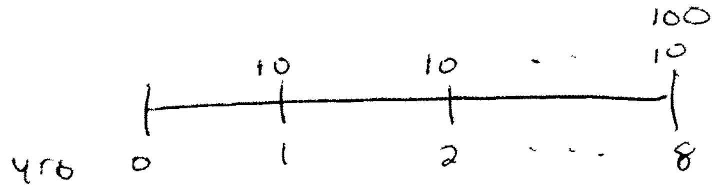
$\therefore I$ = interest amount $= 1560 - 1000 - 300 = 260$

Exp = Exposure $= A + \sum C_t(1-t)$

$$= 1000 + 1000\left(\tfrac{8}{12}\right) - 200\left(\tfrac{6}{12}\right) - 500\left(\tfrac{4}{12}\right) = 1400$$

$$\therefore i_{DW} = \frac{I}{Exp} = \frac{260}{1400} \doteq 18.57\%$$

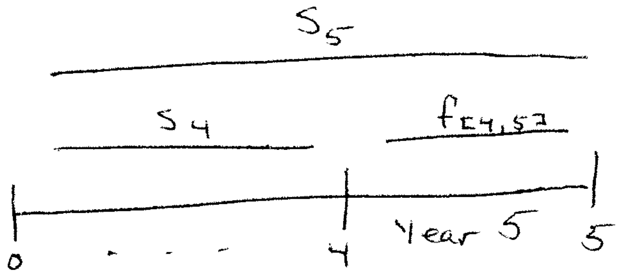
2/ Nov. 2005 Exam FM)



$$MacD = \frac{10(Ia)_{\overline{8|}.08} + 800v^8_{.08}}{10a_{\overline{8|}.08} + 100v^8_{.08}} \doteq 5.989$$

6 / Nov. 2005 Exam FM)

i_k = k-year spot rate = s_k



$s_5 = .075$

$s_4 = .082$

$$(1+s_5)^5 = (1+s_4)^4 \left(1 + f_{[4,5]}\right)$$

$$\Longrightarrow f_{[4,5]} = \frac{(1.075)^5}{(1.082)^4} - 1 \doteq .047$$

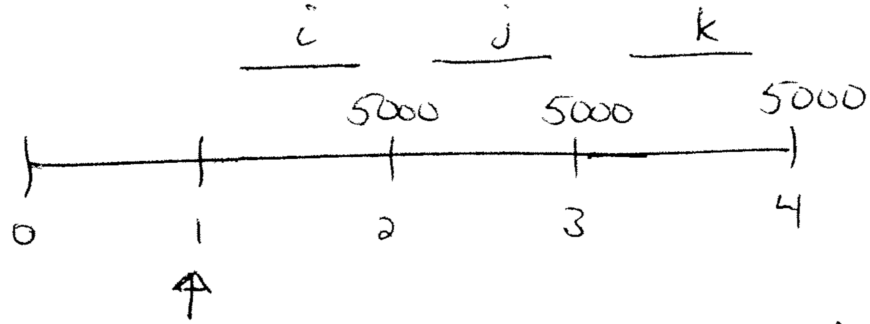
10/ Nov. 2005 Exam FM) The company buys one of the 1-year bond and two of the 2-year bonds.

Prices: P_1 = price of the 1-year bond $= 1000\,v_{.1}$

P_2 = ——————— 2-year bond $= 1000\,v_{.12}^2$

$\therefore$ total cost $= P_1 + P_2 = 1000\,v_{.1} + 2000\,v_{.12}^2 \doteq 2503$

15/Nov. 2005 Exam FM



$$PV = 5000 v_i + 5000 v_j \cdot v_i + 5000 v_k \cdot v_j \cdot v_i$$

See above diagram.

$$i = f_{[1,2]} = \frac{(1+s_2)^2}{1+s_1} - 1 = \frac{(1.0575)^2}{1.05} - 1 \doteq .0651$$

$$j = f_{[2,3]} = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 = \frac{(1.0625)^3}{(1.0575)^2} - 1 \doteq .0726$$

$$k = f_{[3,4]} = \frac{(1+s_4)^4}{(1+s_3)^3} - 1 = \frac{(1.065)^4}{(1.0625)^3} - 1 \doteq .0725$$

$$\therefore PV \stackrel{\underline{VEP}}{=} \frac{5000}{1+i} + \frac{5000}{(1+j)(1+i)} + \frac{5000}{(1+k)(1+j)(1+i)} \qquad (1)$$

$$\doteq \frac{5000}{1.0651} + \frac{5000}{(1.0726)(1.0651)} + \frac{5000}{(1.0725)(1.0726)(1.0651)}$$

$$\doteq 13151.84 \quad \text{(round-off error)}$$

Note: We could get an exact expression for PV by recognizing: $1+i = \frac{(1+s_2)^2}{1+s_1}$, $(1+j)(1+i) = \frac{(1+s_3)^3}{1+s_1}$, and $(1+k)(1+j)(1+i) = \frac{(1+s_4)^4}{1+s_1}$. Then, by (1), we get

$$PV = \frac{5000(1+s_1)}{(1+s_2)^2} + \frac{5000(1+s_1)}{(1+s_3)^3} + \frac{5000(1+s_1)}{(1+s_4)^2} \doteq 13153$$

21/ Nov. 2005 Exam FM) Statements II and III are true

For statement I, we achieve immunization at interest rate i, if using i,

(i) PV (assets) = PV (liabilities)

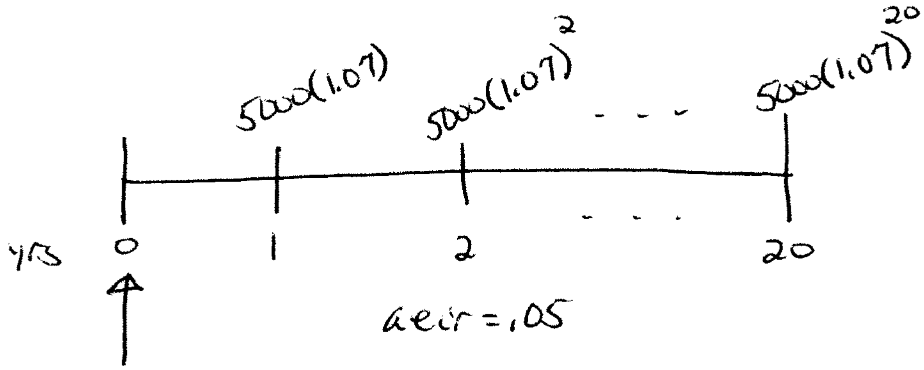
(ii) Duration (assets) = Duration (liabilities)

can be MacD or ModD

(iii) Convexity (assets) > Convexity (liabilities)

Statement I is false.

31 / (Exam FM Sample Questions)

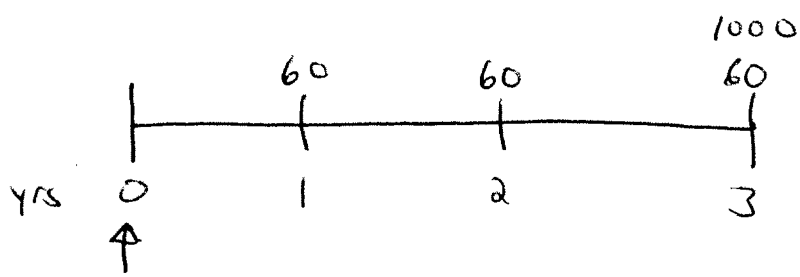


$$PV \stackrel{VEP}{=} 5000(1.07)v + 5000(1.07)^2 \cdot v^2 + \cdots \text{(20 terms)}$$

$$= \frac{5000(1.07)}{1.05}\left(1 + \frac{1.07}{1.05} + \cdots \text{(20 terms)}\right)$$

$$= \frac{5000(1.07)}{1.05} \cdot s_{\overline{20}|\left(\frac{1.07}{1.05}-1\right)} = 122,633.60$$

33 / (Exam FM Sample Questions)



PV = P

$$P = 60v_{.07} + 60v_{.08}^2 + 1060v_{.09}^3 = \frac{60}{1.07} + \frac{60}{(1.08)^2} + \frac{1060}{(1.09)^3}$$

$$\therefore P \doteq 926.03$$

34/ Exam FM Sample Questions) From # 33, we have $P = 926.03$

$$\therefore \; 926.03 = 60\, a_{\overline{3}|i} + 1000\, v_i^3$$

$$\Longrightarrow i \overset{TVM}{=\!=} 8.92\,\%$$

35/ Exam FM Sample Questions) We're given $P(.08) = 100$
and $P'(.08) = -700$

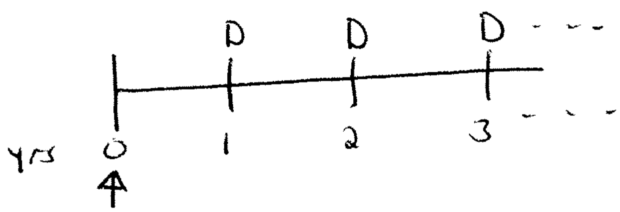
By definition, $\text{Mod}\,D_i = \dfrac{-P'(i)}{P(i)}$

$$\therefore \text{ with } i = .08, \quad \text{Mod}\,D = \frac{-(-700)}{100} = 7$$

Also, $\text{Mod}\,D = v \cdot \text{Mac}\,D \Longrightarrow \text{Mac}\,D = \text{Mod}\,D \cdot (1+i)$

$$\therefore \text{Mac}\,D = 7(1.08) = 7.56$$

36/ Exam FM Sample Questions)

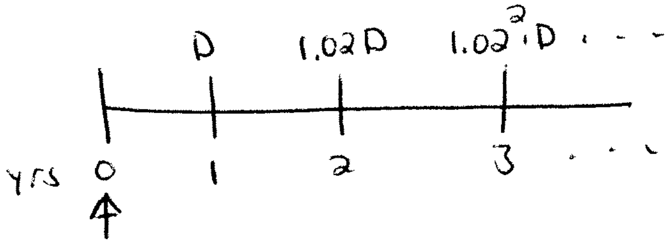


$$\text{Mac}\,D = \frac{Dv + 2Dv^2 + 3Dv^3 + \cdots}{Dv + Dv^2 + Dv^3 + \cdots} = \frac{\cancel{D} \cdot (Ia)_{\overline{\infty}|}}{\cancel{D} \cdot a_{\overline{\infty}|}}$$

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - n v^n}{i} \xrightarrow{n=\infty} (Ia)_{\overline{\infty}|} = \frac{\ddot{a}_{\overline{\infty}|} - 0}{i} = \frac{\frac{1}{d} - 0}{i} = \frac{1}{i \cdot d}$$

$$a_{\overline{\infty}|} = \frac{1}{i} \Longrightarrow \text{Mac}\,D = \frac{1/id}{1/i} = \frac{1}{d} = \frac{1}{i}(1+i) \overset{i=.1}{=\!=} 11$$

37/(Exam FM Sample Questions)



$$\text{Mac}\,D = \frac{Dv + 2(1.02D)v^2 + 3(1.02^2\cdot D)v^3 + \cdots}{Dv + 1.02D\cdot v^2 + 1.02^2\cdot D\cdot v^3 + \cdots}$$

$$= \frac{\cancel{D}(v + 2(1.02)v^2 + 3(1.02)^2\cdot v^3 + \cdots)}{\cancel{D}(v + 1.02v^2 + 1.02^2\cdot v^3 + \cdots)}$$

Let n = numerator and d = denominator

The expression defining n is "georithmic". So use the following trick:

$$n = v + 2(1.02)v^2 + 3(1.02)^2\cdot v^3 + \cdots$$

$$-\left[1.02v\cdot n = \quad (1.02)\cdot v^2 + 2(1.02)^2\cdot v^3 + \cdots\right]$$

$n - 1.02v\cdot n = v + 1.02v^2 + 1.02^2\cdot v^3 + \cdots$ (now it's geometric)

$r = 1.02v$

first term $= v$

$$n(1-1.02v) = \frac{v}{1-1.02v}$$

$$\therefore n = \frac{v}{(1-1.02v)^2}$$

$d = v + 1.02v^2 + 1.02^2 v^3 + \cdots \quad \underset{r=1.02v}{\overset{\text{geometric}}{=}} \quad \dfrac{v}{1-1.02v}$

$$\therefore \text{Mac}\,D = \frac{n}{d} = \frac{v/(1-1.02v)^2}{v/(1-1.02v)} = \frac{1}{1-1.02v} \overset{v=\frac{1}{1.05}}{=} 35$$

51/ Exam FM Sample Questions

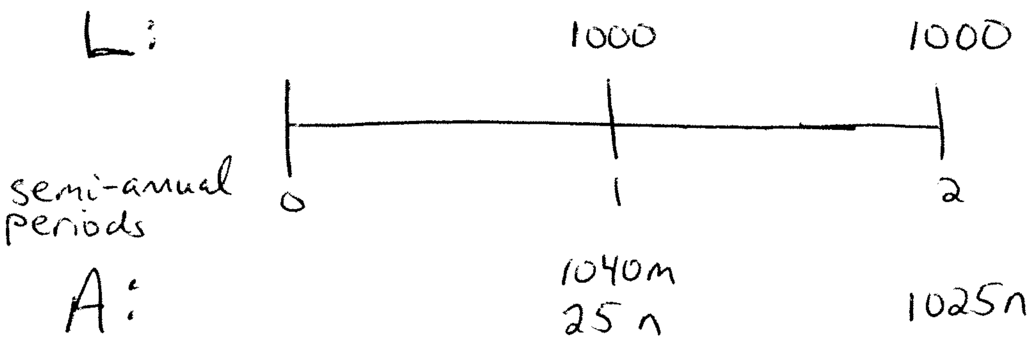
L denotes Liabilities
A denotes Assets

Let m = # of Bond I needed
n = # of Bond II needed

For every Bond I bought, there will be a coupon of 40 and the redemption value of 1000, both paid at the end of six months. Since we have m of them, there will be a payment of $1040 \cdot m$ ~~from~~ paid at the end of six months, from the purchase of the m Bond I's.

Likewise, from the purchase of n Bond II's, there will be a payment of $25n$ at the end of six months and $1025n$ at the end of one year.

The time line is:



Exact matching $\Longrightarrow \begin{cases} 1040m + 25n = 1000 \\ 1025n = 1000 \end{cases}$

$\therefore n = \frac{1000}{1025} \doteq .97561 \Longrightarrow 1040m + 25\left(\frac{1000}{1025}\right) = 1000$

$\Longrightarrow m = .93809$. Buy .93809 Bond I's & .97561 Bond II's

52/Exam FM Sample Questions

The payment from Mortgage I is $X(1.06)$ at $t=1$.

The payment from Mortgage II is R at $t=1$ & $t=2$,

where $Y = R \cdot a_{\overline{2}|.07} \implies R = \dfrac{Y}{a_{\overline{2}|.07}}$

We have

L:		2000	1000
yrs	0	1	2
A:		$X(1.06)$ $Y/a_{\overline{2}|.07}$	$\dfrac{Y}{a_{\overline{2}|.07}}$

$$\therefore \begin{cases} X(1.06) + \dfrac{Y}{a_{\overline{2}|.07}} = 2000 \\ \dfrac{Y}{a_{\overline{2}|.07}} = 1000 \end{cases}$$

$$\implies Y \doteq 1808.02 \quad \& \quad X = 943.40$$

$$\therefore X + Y \doteq 2751.42$$

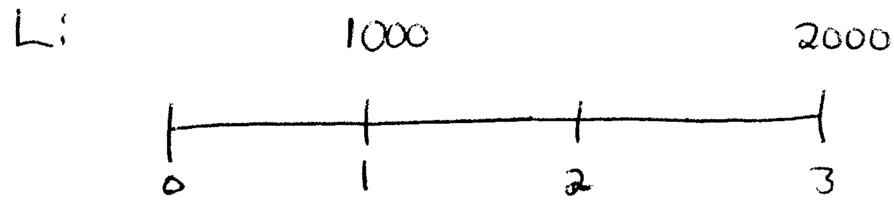
53 / Exam FM Sample Questions

Let m = # of Bond I's needed
and n = ——— II's ———

Each Bond I costs $P^{I} = 1000\,v_{.06}$

Each Bond II costs $P^{II} = 1000\,v_{.07}^{2}$

The timeline is:



A: $1000m \qquad 1000n \xrightarrow{.065} 1000n(1.065)$

$$\therefore \begin{cases} 1000m = 1000 \\ 1000n(1.065) = 2000 \end{cases} \Longrightarrow \begin{matrix} m = 1 \\ n = \frac{2}{1.065} \end{matrix}$$

$\therefore$ total purchase price is $P = P^{I} \cdot m + P^{II} \cdot n$

$$\Longrightarrow P = 1000\,v_{.06}(1) + 1000\,v_{.07}^{2}\left(\frac{2}{1.065}\right) = 2583.66$$

59/Exam FM Sample Questions) We'll do this one by changing the face amounts of the bonds, which is mathematically equivalent to buying different numbers of standard face amount bonds.

Let F_5 denote the face amount of the 5-year bond

and F_{10} ——————————— 10-year —

$PV(L) = 35000\,a_{\overline{15|}}$ $\quad PV(A) = F_5 \cdot v^5 + F_{10} \cdot v^{10}$

We only need to focus on the numerators of the Mac D's. For the liabilities, we have (n_L = numerator for liabilities)

$$n_L = 35000\,(Ia)_{\overline{15|}}$$

For the assets

$$n_A = 5F_5 \cdot v^5 + 10 \cdot F_{10} \cdot v^{10}$$

∴ we solve the system $\quad$ ~~$35a_{\overline{15|}} = F_5 \cdot v^5$~~

$$-\begin{cases} (35000\,a_{\overline{15|}} = F_5 v^5 + F_{10} v^{10})(10) \\ 35000(Ia)_{\overline{15|}} = 5F_5 v^5 + 10F_{10} v^{10} \end{cases}$$

$$\therefore\ 350000\,a_{\overline{15|}} - 35000\,(Ia)_{\overline{15|}} = 5F_5 v^5 \qquad i = .062$$

$\cancel{\Rightarrow F_5}$ (I was about to solve for F_5, but after reading the question, the answer is $F_5 \cdot v^5$.)

The amount invested in 5-year bonds is $F_5 v^5 = \dfrac{350000\,a_{\overline{15|}} - 35000\,(Ia)_{\overline{15|}}}{5}$

$$\therefore\ F_5 v^5 = 208556.21$$

66 / Exam FM Sample Questions) Since the bond is bought at par, $P = 1000$.

<u>Remark</u>: They don't tell you this, but this $P=1000$ is based on an aeir equal to 7.2%, the interest rate that we're "changing from".

$$\Delta P \doteq -P \cdot \text{Mod}D \cdot \Delta i$$

The interest rate changes from .072 to .08

$$\therefore \ \Delta i = .08 - .072 = .008$$

P is the price at the old interest rate of .072

$$\therefore \ P = 1000$$

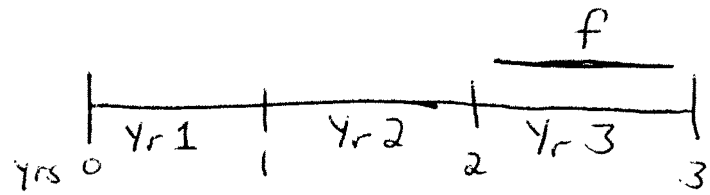
Mod D is the Mod D at the old interest rate of .072

$$\therefore \ \text{Mod}D = v \cdot \text{Mac}D = \frac{7.959}{1.072}$$

$$\therefore \ \Delta P \doteq -1000\left(\frac{7.959}{1.072}\right)(.008) \doteq 59.40$$

$$\Longrightarrow P^{new} = P^{old} + \Delta P \doteq 1000 - 59.40 = 940.60$$

67/ (Exam FM Sample Questions)



$f = f_{[2,3]}$ = third year, one-year forward rate

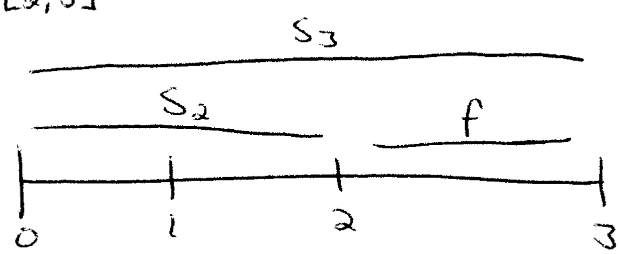
The prices given are prices of zero-coupon bonds with redemption value $= 1$, at the given maturity times. I.e.

$$v_{s_1} = .9542 \implies 1+s_1 = \frac{1}{.9542}$$

$$v_{s_2}^2 = .90703 \implies (1+s_2)^2 = \frac{1}{.90703}$$

$$v_{s_3}^3 = .85892 \implies (1+s_3)^3 = \frac{1}{.85892}$$

We seek $f = f_{[2,3]}$:



$$\therefore\ (1+s_3)^3 = (1+s_2)^2(1+f)$$

$$\implies f = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 = \frac{.90703}{.85892} - 1 \doteq .056$$

68 / Exam FM Sample Questions)

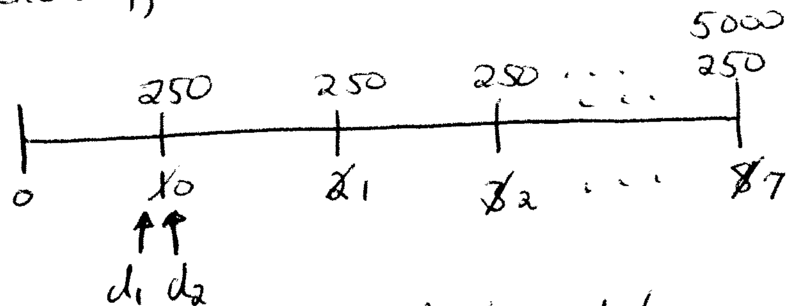
$P = 5000$

$F = C = 5000 \qquad Fr = 250$

$r = .05$

$\therefore \quad 5000 = 250\, a_{\overline{8}|} + 5000 v^8 \implies i \overset{TVM}{=} .05$

Note: Generally, if $P = F = C$ ~~and~~ then $i = r$



Changing the valuation date resets the time values

$$\therefore \; d_1 = \frac{\underbrace{\cancel{250(0)}}_{=0} + 250(1)v + 250(2)v^2 + \cdots + 250(7)v^7 + 5000(7)v^7}{250\, \ddot{a}_{\overline{8}|} + 5000 v^7}$$

$$= \frac{250 (Ia)_{\overline{7}|} + 35000 v^7}{250\, \ddot{a}_{\overline{8}|} + 5000 v^7} \doteq \frac{30378.46}{5250} \doteq 5.78637$$

$$d_2 = \frac{250(1)v + 250(2)v^2 + \cdots + 250(7)v^7 + 5000(7)v^7}{250\, a_{\overline{7}|} + 5000 v^7}$$

$$= \frac{250 (Ia)_{\overline{7}|} + 35000 v^7}{250\, a_{\overline{7}|} + 5000 v^7} = \frac{30378.46}{5000}$$

$$\therefore \quad \frac{d_1}{d_2} = \frac{5000}{5250} \doteq .95238$$

69/(Exam FM Sample Questions)

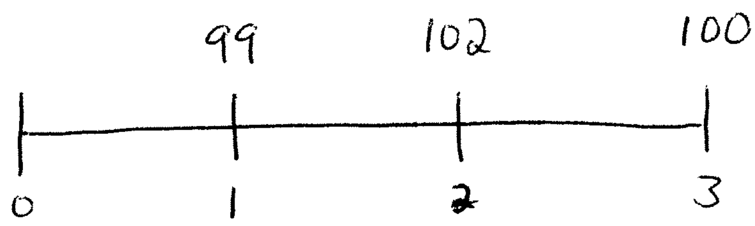
Let m = # of Bond A needed

n = ______ B ______

p = ______ C ______

Then the timeline is:

L:



A:

	1	2	3
	$107m$	$100n$	
	$5p$	$5p$	$105p$

$$\therefore \begin{cases} 107m + 5p = 99 \\ 100n + 5p = 102 \\ 105p = 100 \end{cases}$$

We seek m.

$$p = \frac{100}{105}$$

$$\Rightarrow 107m + 5\left(\frac{100}{105}\right) = 99$$

$$\therefore m \doteq .8807$$

71 / (Exam FM Sample Questions)

	0	2	4	6
L:			6000	
A:		A		B

$i = .05$

Since the liability has an asset before and after, we get full immunization by setting $PV(A) = PV(L)$ and, simultaneously, $\text{Mac}\,D^A = \text{Mac}\,D^L$. Since $PV(A) = PV(L)$ we can just set the numerator of the $\text{Mac}\,D^A$, (n^A) equal to the numerator of the $\text{Mac}\,D^L$, (n^L). We get

$$\begin{cases} 6000\,v^4 = A \cdot v^2 + B \cdot v^6 \\ 6000(4) \cdot v^4 = 2Av^2 + 6Bv^6 \end{cases}$$

We can eliminate B by multiplying the first equation by 6 and subtracting the second equation, getting

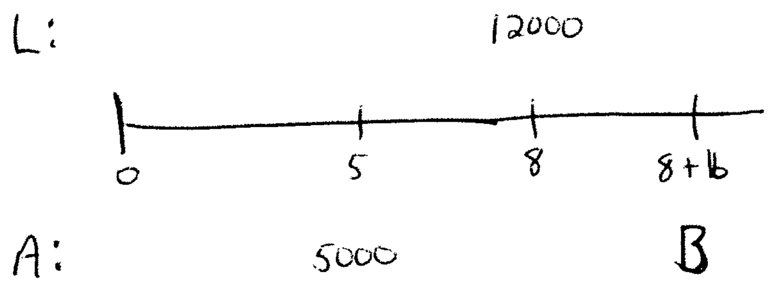
$$12000\,v^4 = 4Av^2 \implies A = 3000\,v^2_{.05}$$

Then $6000v^4 = (3000v^2) \cdot v^2 + Bv^6$

$$\implies B = 3000(1.05)^2$$

$$\therefore |A - B| = 586.41$$

72 / Exam FM Sample Questions



$i = .03$

$$\begin{cases} PV(L) = PV(A) \\ \Lambda^L = \Lambda^A \end{cases}$$ ← see notation from previous problem

$$12000 v^8 = 5000 v^5 + B \cdot v^{8+b}$$

$$12000(8) v^8 = 5000(5) v^5 + B(8+b) v^{8+b}$$

Substitution is easier to use in this problem. From the first equation $Bv^{8+b} = 12000 v^8 - 5000 v^5$. ★

Substituting into the second equation yields

$$96000 v^8 = 25000 v^5 + (12000 v^8 - 5000 v^5)(8+b)$$

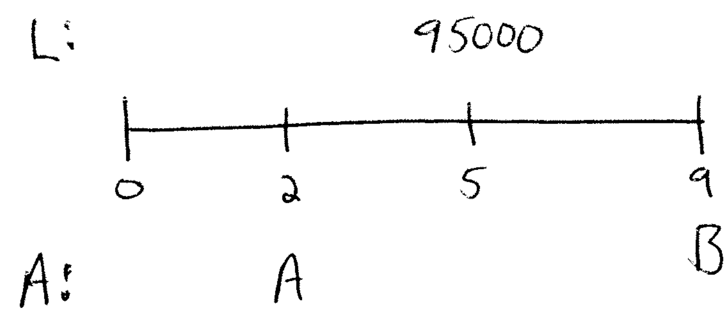
$$\Longrightarrow b = \frac{96000 v^8 - 25000 v^5}{12000 v^8 - 5000 v^5} - 8 \doteq 2.50765$$

Then, from ★, $Bv^{8+b} = 12000 v^8 - 5000 v^5$

$$\Longrightarrow B \doteq 7039.27$$

$$\therefore \frac{B}{b} \doteq 2807$$

73/ Exam FM Sample Questions)



$i = .04$

$$\begin{cases} PV(L) = PV(A) \\ n^L = n^A \end{cases}$$

$$(95000 v^5 = A v^2 + B v^9)(9)$$

$$- \quad 95000(5) v^5 = 2A v^2 + 9B v^9$$

$$380000 v^5 = 7A v^2 \implies A = \frac{380000 v^3}{7}$$

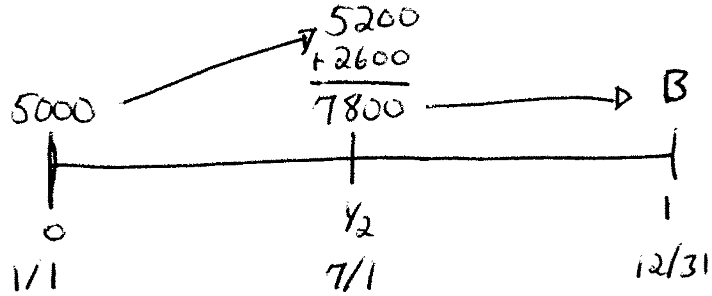
Then $95000 v^5 = \frac{380000 v^3}{7} \cdot v^2 + B v^9$

$$\implies B = \frac{285000 (1.04)^4}{7}$$

$$\therefore \frac{A}{B} = \frac{380000}{285000 (1.04)^7} \doteq 1.0132$$

Remark: If you don't want to get exact values, then just approximate A & B to get $\frac{A}{B}$

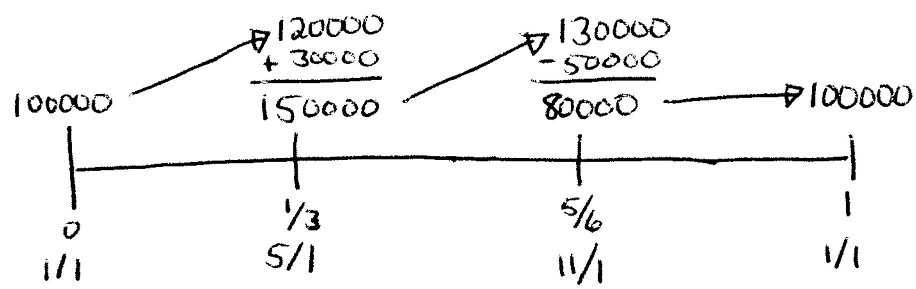
78/ (Exam FM Sample Questions)



$$B = 5000(1.09) + 2600(1.09)^{1/2} = 8164.48$$

$$i = i_{TW} \implies 1+i = \frac{5200}{5000} \cdot \frac{B}{7800} \implies i \doteq .0886$$

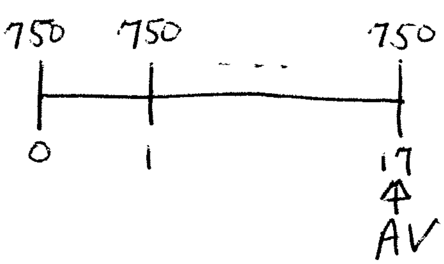
83/ (Exam FM Sample Questions)



$$i = i_{TW} \implies 1+i = \frac{120000}{100000} \cdot \frac{130000}{150000} \cdot \frac{100000}{80000} \implies i \doteq 30\%$$

89/ (Exam FM Sample Questions)

Deposits:



$AV(\text{deposits}) = 750\, s_{\overline{18}|.07}$

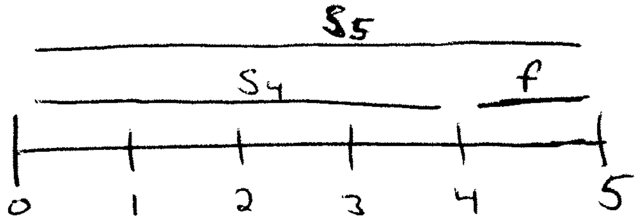
Tuition costs for the 18^{th} school year $= T_{18} = 6000(1.05)^{17}$

The excess at $t=17$ is $E = 750\, s_{\overline{18}|.07} - 6000(1.05)^{17}$

$T_{19} = 6000(1.05)^{18}$. The excess accumulates to $E(1.07)$

$\therefore X = 6000(1.05)^{18} - E(1.07) = 1870.25$

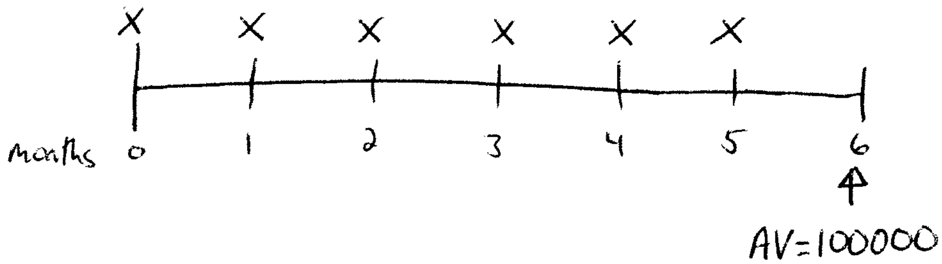
92/ Exam FM Sample Questions



We seek $f = f_{[4,5]}$

$$(1+s_4)^4(1+f) = (1+s_5)^5 \Longrightarrow f = \frac{(1.095)^5}{(1.09)^4} - 1 \doteq 0.115$$

111/ Exam FM Sample Questions



The X at $t=0$ accumulates to $X(1+s_{0.5})^{0.5}$. However, since each bond is zero coupon, $P = C \cdot v^k$, or $\frac{P}{C} = v^k$, which is what's given in the table. E.g. the 94% in the table means $.94 = $ sdf using the 6-month spot rate. Then the saf $= (1+s_{0.5})^{0.5} = \text{sdf}^{-1} = (.94)^{-1}$.

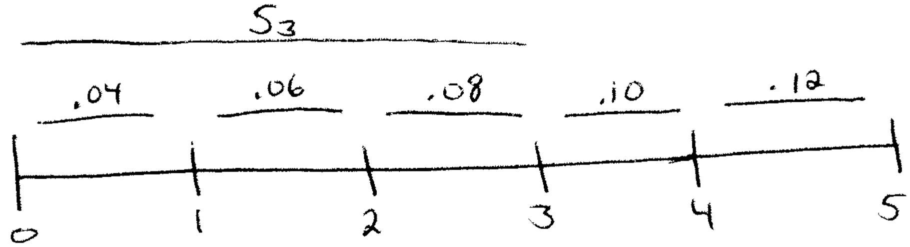
$\therefore$ The X at $t=0$ accumulates to $X(.94)^{-1}$

We should techically use forward rates to accumulate the X's at other times. However, since "bond prices will not change during the 6-month period", we can use the given spot rate information. E.g. the X at time 3 accumulates to $X \cdot {}_3\text{af} = X \cdot ({}_3\text{df})^{-1} = X(.97)^{-1}$. Using this logic, we have

$$100000 = X\left[(.94)^{-1} + (.95)^{-1} + (.96)^{-1} + (.97)^{-1} + (.98)^{-1} + (.99)^{-1}\right]$$

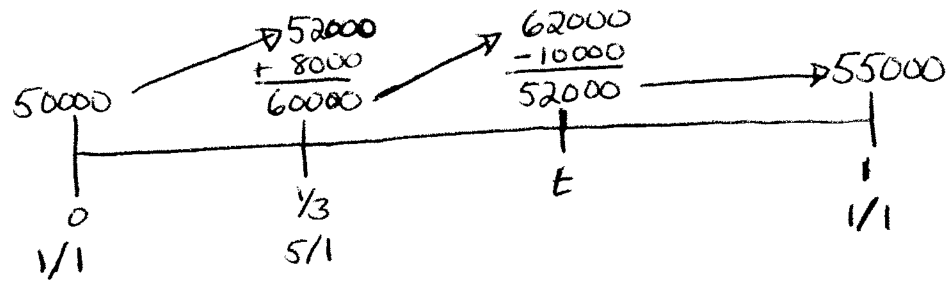
$$\Longrightarrow X \doteq 16078.29$$

119 / Exam FM Sample Questions



$$(1+S_3)^3 = (1.04)(1.06)(1.08) \Longrightarrow S_3 = .059874$$

120 / Exam FM Sample Questions



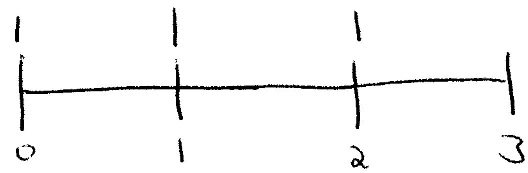
$$1+i_{TW} = \frac{52000}{50000}\cdot\frac{62000}{60000}\cdot\frac{55000}{52000} \Longrightarrow i_{TW} = .13\bar{6} \left(= i_{DW}\right)$$

$$i_{DW} = .13\bar{6} \Longrightarrow 55000 = 50000(1+.13\bar{6}) + 8000(1+.13\bar{6}(2/3)) - 10000(1+.13\bar{6}(1-t))$$

$$\Longrightarrow t = .5886$$

121 / Exam FM Sample Questions

Annuity A:

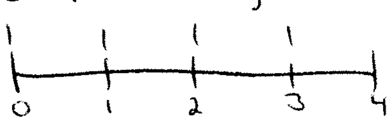


$$MacD^A = \frac{1(0)+1(1)v+1(2)v^2}{1+v+v^2} \doteq \frac{v+2v^2}{1+v+v^2} = .93$$

$$\Longrightarrow v+2v^2 = .93+.93v+.93v^2 \Longrightarrow 1.07v^2 + .07v - .93 = 0$$

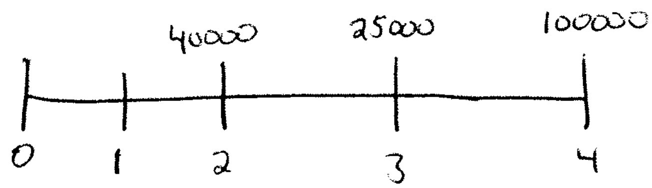
$\therefore$ by quadratic formula, $v \doteq .90015$

Annuity B:



$$MacD^B = \frac{1(0)+1(1)v+1(2)v^2+1(3)v^3}{1+v+v^2+v^3} = \frac{v+2v^2+3v^3}{1+v+v^2+v^3} \overset{v\doteq .90015}{=\!=\!=} 1.369$$

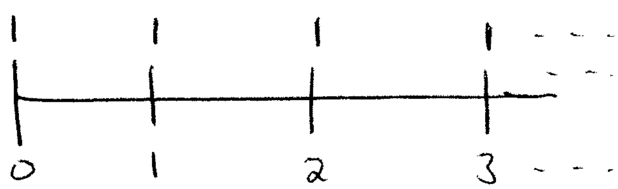
122 / Exam FM Sample Questions



$i = .07$

$$\text{MacD} = \frac{40000(2)v^2 + 25000(3)v^3 + 100000(4)v^4}{40000v^2 + 25000v^3 + 100000v^4} \doteq 3.314$$

124 / Exam FM Sample Questions



$$\text{MacD} = 30 = \frac{\overset{=0}{1(0)} + 1(1)v + 1(2)v^2 + 1(3)v^3 + \cdots}{1 + v + v^2 + v^3 + \cdots} = \frac{v + 2v^2 + 3v^3 + \cdots}{\underbrace{1 + a_{\overline{\infty}|}}_{= \ddot{a}_{\overline{\infty}|}}}$$

Remarks: 1) The numerator is $(Ia)_{\overline{\infty}|} = \frac{P}{i} + \frac{Q}{i^2}$ w/ $P = Q = 1$

I.e. the numerator equals $\frac{1}{i} + \frac{1}{i^2} = \frac{1+i}{i^2}$

2) The denominator equals $1 + \frac{1}{i} = \frac{1+i}{i}$ ← same

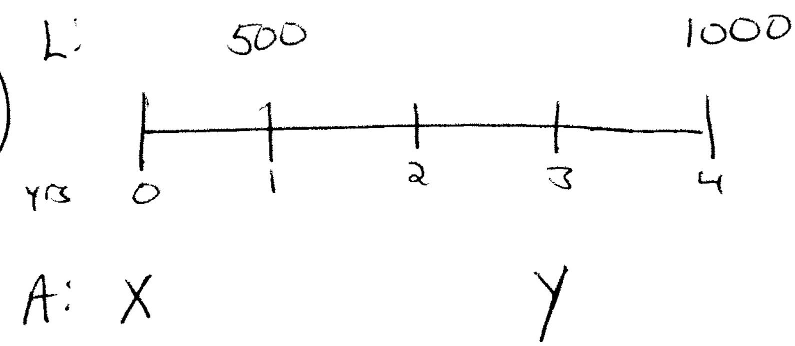
(If you're thinking $\ddot{a}_{\overline{\infty}|}$, then it's $\ddot{a}_{\overline{\infty}|} = \frac{1}{d} = \frac{1}{i/(1+i)} = \frac{1+i}{i}$)

$$\therefore \text{MacD} = 30 = \frac{\left(\frac{1+i}{i^2}\right)}{\left(\frac{1+i}{i}\right)} = \frac{1}{i} \implies i = \frac{1}{30}$$

$$\text{ModD} = v \cdot \text{MacD} = \frac{1}{1 + 1/30}(30) \doteq 29.03$$

127 / Exam FM Sample Questions

aeir = .10



$$\begin{cases} PV(L) = PV(A) \\ \cancel{MacD^L = MacD^A} \\ n^L = n^A \end{cases} \implies \begin{aligned} 500v + 1000v^4 &= X + Yv^3 \\ 500v + 4(1000)v^4 &= 0(X) + 3Yv^3 \end{aligned}$$

$$\therefore Y = \frac{500(1.1)^2 + 4000v}{3} \doteq 1413.79$$

and $X \doteq 75.36$ (Either A or B is correct.)

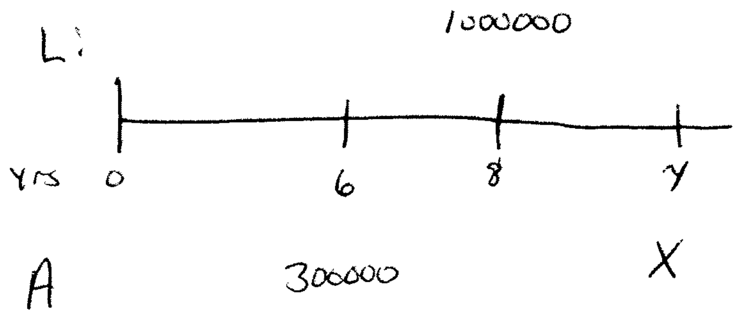
The easiest way to proceed is to change the interest rate a small amount, to say 12% (±2% is good), and see how this changes the PV(L) vs PV(A). Be sure to use exact values for X & Y, and not rounded values. At $i = .12$ we get

$$PV(L) = 500v_{.12} + 1000v_{.12}^4 = 1081.94665$$

$$PV(A) = X + Yv_{.12}^3 = 1081.665438$$

Since $PV(L) > PV(A)$, even though by not much, immunization has not be achieved

128 / (Exam FM Sample Questions)



$i = .04$

$$\begin{cases} PV(L) = PV(A) \\ n^L = n^A \end{cases} \implies \begin{matrix} 1000000\, v^8 = 300000 v^6 + X \cdot v^y \\ 8(1000000) v^8 = 6(300000) v^6 + y \cdot X v^y \end{matrix}$$

Using substitution, $X \cdot v^y = 1000000 v^8 - 300000 v^6$

$$\xrightarrow[\text{second equation}]{\text{from}} \quad y = \frac{8000000 v^8 - 1800000 v^6}{1000000 v^8 - 300000 v^6} \doteq 8.96068$$

Then $X = (1000000 v^8 - 300000 v^6)(1.04)^y \doteq 701,458.26$

129 / (Exam FM Sample Questions)

$PV(L) = PV(A)$

$\text{Mac } D^L = \text{Mac } D^A$

$i = .07$

Since $PV(L) = 573v^2 + 701v^5 \doteq 1000$, and noting that $PV(A) = \text{Price}^{(\text{Total})}$, we can eliminate answer choices C & D since in both cases $PV(A) = 1274$.

$$\text{Mac } D^L = \frac{2(573)v^2 + 5(701)v^5}{573v^2 + 701v^5} \doteq 3.5$$

Note that ~~for char~~ 3.5 is halfway between 1 & 6.

For choice B, $\text{Mac } D^{\text{Portfolio}} = \frac{572}{1000}(1) + \frac{428}{1000}(6)$

We could do the arithmetic, or notice the weight on 1 is greater than the weight on 6, and so Mac D is closer to 1 than to 6. I.e. Mac D < 3.5 (Eliminate B)

~~Answer choice E is worded oddly. I think they mean you have invested in a~~

★ Since answer choice A has an asset before the first liability, and after the last one, if $\text{Mac } D^A = 3.5$, then it is our answer. The redemption values are: Bond A – $500(1.07)$ Bond B – $500(1.07)^6$.

$$\text{Mac } D = \frac{535v + 500(1.07)^6 \cdot (6)v^6}{535v + 500(1.07)^6 \cdot v^6} = \frac{3500}{1000} = 3.5. \quad \text{Answer} = A$$

130 / Exam FM Sample Questions

L:		402.11	402.11	402.11
	0	1	2	3
A:		X		Y

$i = .10$

$$\begin{cases} PV(L) = PV(A) \\ n^L = n^A \end{cases} \Longrightarrow \begin{aligned} 402.11\, a_{\overline{3}|.10} &= Xv + Yv^3 \\ 402.11\, (Ia)_{\overline{3}|.10} &= Xv + 3Yv^3 \end{aligned}$$

Subtracting first equation from the ~~second~~ second yields:

$$402.11\left((Ia)_{\overline{3}|.1} - a_{\overline{3}|.1}\right) = 2Yv_{.1}^3$$

$$\Longrightarrow Y \doteq 623.27$$

$$\therefore\ X = \left(402.11\, a_{\overline{3}|} - Yv^3\right)(1.1) \doteq 584.88$$

$\therefore$ 1-year bond costs $X \cdot v \doteq 531.72$

and 3-year bond costs $Y \cdot v^3 \doteq 468.27$

131 / Exam FM Sample Questions — The easiest way to proceed is as follows:

1) $PV(A) = PV(L) = 9697$ (This does not eliminate any answer choices, as you can check.)

2) $MacD^A = MacD^L = 15.24$

<u>Note</u>: For a k-year zero coupon bond, $MacD = k$

Check answer choices for $MacD^A$

(A) $MacD = \frac{3077}{9697}(5) + \frac{6620}{9697}(20) \doteq 15.24$

(B) $MacD = \ldots$ (or notice the weight on 5 is $\frac{6620}{9697}$, larger than 0.5. So the weighted average of 5 & 20 is closer to 5, and so can not equal 15.24.) Eliminate (B)

(C) MacD is closer to 20 than 15, so $MacD \neq 15.24$
Eliminate (C)

(D) Same as (C). Eliminate D.

(E) $MacD = \frac{9232}{9697}(15) + \frac{465}{9657}(20) \doteq 15.24$

Answer is either (A) or (E)

3) $\cancel{MacD^A > MacD^L}\ MacC^A > MacC^L$

Check answer choices (A) & (E)

(A) $MacC^A = \frac{3077}{9697}(5)^2 + \frac{6620}{9697}(20)^2 \doteq 281 > MacC^L$

$\therefore$ Answer $= (A)$

Remark, For (E), $MacC^A = \frac{9232}{9697}(15)^2 + \frac{465}{9697}(20)^2 \doteq 233 < MacC^L$

132 / Exam FM Sample Questions

$20000(1.1) = 22000$

$\therefore$ The customer withdraws 11000 at $t=1$. Then $11000(1.1) = 12100$ is the withdrawal at $t=2$.

Let X = face amount (redemption value) for Bond H

Y = ________________________________ I

Z = ________________________________ J

Note: P^H = Price of Bond H $= X v_{.1}$

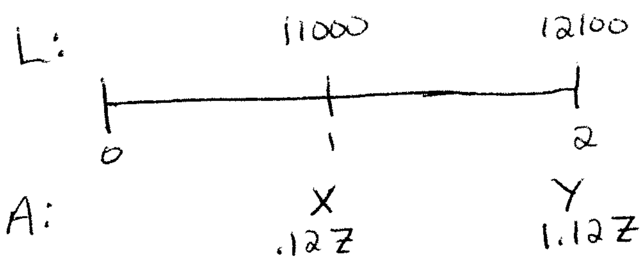
P^I = ______________ I $= Y v_{.11}^2$

P^J = ______________ J $\underline{\text{"sells at par"}}$ Z

You can see this also as follows

$$P^J = .12 Z a_{\overline{2}|.12} + Z v_{.12}^2 = Z(\overbrace{.12 a_{\overline{2}|.12} + v_{.12}^2}^{=1}) = Z$$

The time line is:



$$\therefore \begin{cases} 11000 = X + .12Z \\ 12100 = Y + 1.12Z \end{cases}$$

$$P = P^H + P^I + P^J = \frac{X}{1.1} + \frac{Y}{(1.11)^2} + Z$$

Highest Profit $\Longrightarrow$ $\underbrace{\text{smallest price for bonds}}$

One way to proceed is as follows: Consider P for each answer choice.

(A) $P = 9091 + 8264 + 2145 = 19500$ $\xrightarrow{\text{Start here}}$ $X = 9091(1.1) = 10000$, $Y = 8264(1.11)^2 \doteq 10182$, $Z = 2145$ } at $t=1$, provides $X + .12Z = 10257.4$ (not enough; move to next lowest P.)

(B) $P = 10000 + 10000 = 20000$

(C) $P = 19821$

(D) $P = 19641$

(E) $P = 19625$ $\Longrightarrow$ $X = 9703$ } at $t=1$, provides $X + .12Z = 11000$; $Z = 10804$ } at $t=2$, provides $1.12Z = 12100$ ✓

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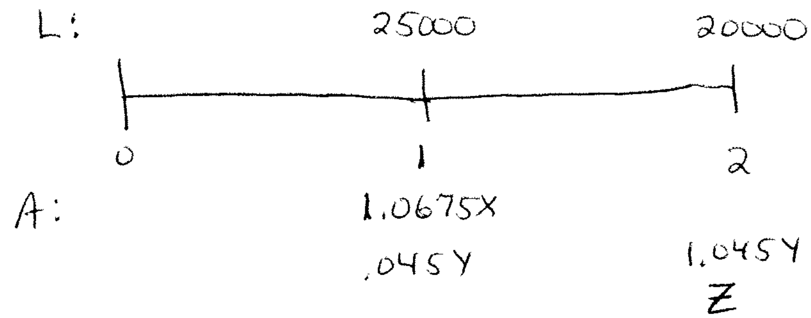
Let X = face amount of bond in part i)
Y = ———————————————— ii)
Z = ———————————————— iii)

The timeline is:



$$\therefore \begin{cases} 25000 = 1.0675X + .045Y \\ 20000 = \qquad\qquad 1.045Y + Z \end{cases}$$

Remarks: 1) There are infinitely many solutions to this system

2) We seek the ~~value~~ solution that gives the smallest discounted value of assets, which is obtained by discounting at the largest interest rates.

3) For parts i) & ii), the "at par" means the bonds are bought at par. So $F = C = P$, and so $r = i$. I.e. the bond in i) yields $i = .0675$ and the bond in ii) yields $i = .045$. The bond in iii) yields $i = .05$, as given.

From remarks 2) & 3), do not use the bond in ii). I.e. take $Y = 0$. We can solve for X & ~~Y~~ Z, or just recognize we'll be discounting the 25000 using $i = .0675$ and the 20000 using $i = .05$, obtaining $\frac{25000}{1.0675} + \frac{20000}{(1.05)^2} = 41559.79$